Unsteady Turbulent Boundary Layers and Separation

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The time dependent turbulent boundary-layer equations are integrated numerically with a two-layer eddy viscosity model (Cebeci-Smith formulation), for transient or oscillatory outer flows. Comparisons with previous theoretical results indicate that the present method is at least as good as the others. Extensive comparisons with experimental data also are attempted for the first time. It appears that in a certain range of frequencies the agreement is satisfactory. Further, some characteristic quantities, like the mean velocity profile or the wall shear phase angle, are predicted accurately but other properties, like the averaged fluctuations of the velocity, indicate some discrepancies. The present method also is capable of integrating past the point of zero skin friction and into regions of partially reversed flow. The phenomenon of separation in unsteady flow also is investigated.

I. Introduction

MOST of the engineering problems in aerodynamics involve unsteady phenomena; for example, the unsteady stall of airfoils, the problem of flutter, the aerodynamics of a helicopter rotor, and certainly flows through cascades of turbomachinery often introduce dynamic phenomena that cannot be described by quasisteady models. Many of these problems have been considered up to now only via inviscid theories, even through the effects of viscosity have proved to play a vital role. The complexity of the problem is enhanced because, in most cases, the boundary layers are turbulent and separation phenomena are involved.

Nevertheless, it appears necessary now to investigate more throroughly unsteady turbulent boundary layers, even though the steady problem is not yet entirely understood. Another reason rendering such efforts rather premature is that there is very little experimental information available. In fact, such efforts may even appear dubious, since all of the approximate methods rely on the determination of some arbitrary constants by comparison with experiments. However, it appears that experimentalists often are guided by some theoretical predictions to look for a specific phenomenon or measure a particular quantity. Further, the urgent need for some engineering estimates has resulted in some foolhardy attempts to extend the established methods to time-dependent flows. The engineering problem of calculating turbulent shear flows is centered around the Reynolds stress and the various closure assumptions that permit its functional dependence on mean quantities. There are various models in use today,[1-5] some based on the mixing length concept, others on the turbulent energy equation, etc., but in the present paper we will refer to and compare the relative success only of those that have been extended to unsteady flows.

Numerical calculations of time-dependent laminar flows have preceded the calculations of turbulent flows.[6-9] For turbulent boundary layers, Bradshaw[10] employed the turbulent energy equation and Cebeci and Keller[11] a generalized eddy viscosity concept, but in both publications one space coordinate was eliminated to render the problem two-dimensional.

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Abbott and Cebeci[12] also considered time-dependent turbulent flows over an infinite plate. McDonald and Shamroth[13] employed an integral method and a mixing length model and later Shamroth and Kreskovsky[14] used the turbulent kinetic energy equation in an extension to the method of McDonald and Camarata.[15] Patel and Nash,[16] and later Nash et al.[17] and Singleton et al.,[18] have extended the turbulent energy method for closing the system of differential equations to calculate transient and oscillatory turbulent flows with or without pressure gradients.

Dwyer[19] also has developed a technique based on the Cebeci-Smith formulation and integrated the boundary-layer equations by a finite-difference method. McCroskey and Philippe[20] later utilized this method, checked it against previous theoretical and experimental results, and calculated the flowfields about airfoils. Kuhn and Nielsen[21] also extended their integral method technique to unsteady flow.

The authors will not attempt here an overall account of shortcomings of the aforementioned techniques, but will briefly mention the following facts. Integral methods are based on a successful assumption for velocity profiles and, as a result, are incapable of predicting in-phase and out-of-phase mean velocity disturbances. etc. Such methods also have been proved to be inaccurate in predicting the flow near separation. Methods based on the mixing length concept do not take into account the history of turbulence, and are based on the hypothesis that the Reynolds stress depends entirely on the local value of the mean velocity and its derivatives. Methods based on the turbulent kinetic energy equation usually require the omission of the laminar shear stress in order to render the set of equations hyperbolic. Thus details of the flow near the wall may be missed, and perhaps the flow near separation may be misinterpreted. All of the preceding arguments, of course, are qualitative, and only a thorough comparison with experimental data can indicate the most accurate and effective method, if any. Unfortunately, there is very little experimental evidence to date,[22,23] and it appears that very few of the aforementioned investigators have attempted to compare their predictions with the available experimental data. As will be shown in this paper, there is a strong indication that all of the theoretical methods, including the present, fail to predict some of the characteristics features of oscillatory flow.

For laminar flow it was proposed,[24] and later verified numerically[9,25] and experimentally,[26] that the vanishing of the wall shear is not related to separation. The present paper extends these arguments to turbulent flow and proposes a method of integration through regions of partially reversed flow. Separation, as defined in Refs. 24 and 27 and previously studied for steady turbulent flow,[28,29] is calculated for a transient motion that involves an ever steepening adverse pressure gradient.

II. Governing Equations and Method of Solution

The boundary-layer equations for two-dimensional incompressible, unsteady, turbulent flow are

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\partial U_e}{\partial t} - U_e\frac{\partial U_e}{\partial x} = \frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{2}$$

where x, y are the coordinates and u, v are the average velocity components, along and perpendicular to the wall, respectively, t is the time, ρ is the density, U_e is the freestream velocity, and τ is the shear stress given by

$$\tau = \mu(\partial u/\partial y) - \rho\overline{u'v'} \tag{3}$$

It is tacitly assumed here that the spectrum of frequencies of the random fluctuations, as well as the discrete random frequencies that appear to persist for a small period of time, are one order of nagnitude larger than the frequency of the outer flow oscillation, or the inverse of the time scale of the transiency of the outer flow. This would permit an averaging over the small time scale of the random phenomena to eliminate the turbulent fluctuations, except of course for the nonlinear contribution of the Reynolds stress, and leave the averaged mean flow to be a function of time. In Eq. (3), μ is the coefficient of viscosity, u' and v' are the fluctuating turbulent velocity components, and the overbar denotes time averaging. With the introduction of the eddy viscosity

$$\epsilon^* = -\rho\,\overline{u'v'}/(\partial u/\partial y) \tag{4}$$

Eq. (3) assumes the form

$$\tau = \mu\bar{\epsilon}(\partial u/\partial y) \tag{5}$$

where $\bar{\epsilon}$ is a nondimensional "total" viscosity, incorporating both the molecular and apparent turbulent viscosities,

$$\bar{\epsilon} = 1 + \epsilon = 1 + \epsilon^*/\mu \tag{6}$$

Taking Eq. (5) into account, the system of Eqs. (1) and (2) is replaced by the system

$$\partial u/\partial x + \partial u/\partial y = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\partial U_e}{\partial t} - U_e\frac{\partial U_e}{\partial x} = \nu\frac{\partial}{\partial y}\left(\bar{\epsilon}\frac{\partial u}{\partial y}\right) \tag{8}$$

where $\nu = \mu/\rho$ is the kinematic viscosity.

Let L be a representative length, U_∞ a representative constant velocity of the time-dependent freestream velocity, and $Re = U_\infty L/\nu$, the Reynolds number. A modification of Görtler's transformation now is introduced with new independent variables

$$\xi = \frac{1}{LU_\infty}\int_0^{x/L} U_{em}(x)\,\mathrm{d}x$$

$$\eta = \frac{(Re)^{1/2}}{LU_\infty}$$

$$U_{em}(x)\,(2\xi)^{-1/2}y, \quad \tau = \frac{U_\infty}{L}\,t \tag{9}$$

and new dependent variables

$$F = \frac{u}{U_e}, \quad V = (Re)^{1/2}\frac{(2\xi)^{1/2}}{U_{em}}v + \frac{U_e}{U_{em}}\eta(\beta_m - 1)F \tag{10}$$

for the velocity components in the ξ and η directions, respectively. The quantities $\beta_m = (2\xi/U_{em})\,(\partial U_{em}/\partial\xi)$, $\beta = (2\xi/U_e)(\partial U_e/\partial\xi)$ are pressure gradient parameters independent and dependent on time, respectively. The quantity U_{em} is a representative distribution of the outer inviscid flow. In the present calculations, a time average, $U_{em}(x) = 1/T\int_0^T U_e(x,t)\,\mathrm{d}t$, has been used and for transient flow, the initial distribution $U_{em}(x) = \mathrm{H}_e(x,0)$.

In terms of the new dependent and independent variables, Eqs. (7) and (8) become

$$B_1\partial F/\partial\xi + B_2F + \partial V/\partial\eta = 0 \tag{11}$$

where

$$B_1 = 2\xi\frac{U_e}{U_{em}}, \quad B_2 = \frac{2\xi}{U_{em}^2}\left[LU_\infty\left(\frac{\partial U_e}{\partial x} - \frac{U_e}{U_{em}}\frac{\partial U_{em}}{\partial x}\right) + \frac{U_eU_{em}}{2\xi}\right] \tag{12}$$

and

$$\frac{\partial^2 F}{\partial\eta^2} + A_1^*\frac{\partial F}{\partial\eta} + A_2^*F + A_3^* + A_4^*\frac{\partial F}{\partial\xi} + A_5^*\frac{\partial F}{\partial\tau} = 0 \tag{13}$$

where

$$A_1^* = (1/\bar{\epsilon})\,[-V + (\partial\bar{\epsilon}/\partial\eta)] \tag{14a}$$

$$A_2^* = -\frac{1}{\bar{\epsilon}}\left(LU_\infty\frac{2\xi}{U_{em}^2U_e}\frac{\partial U_e}{\partial t} + \beta\frac{U_e}{U_{em}}F\right. \tag{14b}$$

$$A_3^* = \frac{1}{\bar{\epsilon}}LU_\infty\frac{2\xi}{U_{em}^2U_e}\frac{\partial U_e}{\partial t} + \beta\frac{U_e}{U_{em}} \tag{14c}$$

$$A_4^* = -(2\xi/\bar{\epsilon})\,(U_e/U_{em})F \tag{14d}$$

$$A_5^* = U_\infty^2\,(2\xi/\bar{\epsilon}U_{em}^2) \tag{14e}$$

Equation (13) is brought again[9] in the form of a steady-state equation by introducing a difference form of the time derivative $\partial F/\partial\tau \simeq (F - F^\circ)/\Delta\tau$ where $F^\circ = F\ (\tau - \Delta\tau)$. Equation (13) then takes the form

$$\partial^2F/\partial\eta^2 + A_1\partial F/\partial\eta + A_2F + A_3 + A_4\partial F/\partial\xi = 0 \tag{15}$$

where

$$A_1 = A_1^* \quad A_2 = A_2^* + (A_5^*/\Delta\tau)$$

$$A_3 = A_3^* - (A_5^*/\Delta\tau)F^\circ \quad A_4 = A_4^* \tag{16}$$

The nonlinear differential equation [Eq. (15)], together with Eq. (11), is solved numerically using subroutines originally developed by Flügge-Lotz Blottner and Davis (see Ref. 9). An upwind differencing scheme is employed for integration through regions of reversed flow.[9] More details about the numerical integration of time dependent flows and in particular oscillatory flows can be found in Refs. 30-32.

III. Eddy Viscosity Models

The turbulent boundary layer is regarded as a composite layer consisting of an inner and an outer layer. In the inner layer an expression based on Prandtl's mixing length is used. That is

$$\epsilon_i = \epsilon_i^*/\mu = (\rho\ell^2/\mu)\,|\partial u/\partial y| \tag{17}$$

where the mixing length ℓ is assumed to have the form

$$\ell = k_\ell y[1 - \exp(-y/A)] \tag{18}$$

In the previous equation k_ℓ is a constant, empirically calculated and traditionally given the value 0.41, and A is the van Driest[33] damping factor. This factor was later generalized by Cebeci[34] to include the effects of pressure gradient and blowing, but various authors have questioned the effectiveness of this generalization for the case of zero blowing. In the present study we assumed

$$A = 26\nu(\,|\tau_w|/\rho)^{-1/2} \tag{19}$$

where τ_w is the wall shear. The present authors have tested this model for steady flow against classical experimental data to check the necessity of the inclusion of the pressure terms in the definition of A.[29] Similar numerical experiments were performed for unsteady flow to test the effect of the pressure term $U_e \partial U_e/\partial x$, which now is augmented by a dynamic term, $\partial U_e/\partial t$, on the averaged profile. At least for the cases considered, it appears that the effect is negligible. The reader should also notice that only the absolute value of the wall shear τ_w appears in Eq. (19). The present model is expected to hold beyond the point of zero wall shear, in a region of partially reversed flow. The exponential appearing in Eq. (18), therefore, always should represent a damping effect due to the presence of the wall. Notice further that the eddy viscosity as given by Eq. (17) is a coefficient appearing on the right-hand side of Eq. (8), and should represent the turbulent diffusion of vorticity, which results in flow decceleration for either a positive or negative value of the local mean velocity gradient. All of the preceding are straightforward assumptions which eventually can be justified only by comparison with experiments.

In the outer region, the velocity defect law is assumed[35] and the eddy viscosity reads

$$\epsilon_0 = \epsilon_0^*/\mu = (\rho k_2 U_e/\mu)\,\delta^* \bar{\gamma} \tag{20}$$

where U_e is the outer flow velocity, δ^* is the displacement thickness, k_2 is another empirical constant equal to 0.0168, and $\bar{\gamma}$ is an intermittency factor[35] given by

$$2\bar{\gamma} = 1 - \operatorname{erf}\,[5(y/\delta - 0.78)] \tag{21}$$

with δ the boundary-layer thickness.

In terms of the stretched and dimensionless quantities, the eddy viscosity for the inner and outer regions, respectively, reads

$$\bar{\epsilon}_i = 1 + R_e^{1/2}\,\frac{U_e}{U_{em}}\,k_i^2 \left\{1 - \exp\left[-\frac{(2\xi R_e)^{1/4}}{26}\,\frac{Ue}{U_{em}}\left|\frac{\partial F}{\partial \eta}\right|_w\right)^{1/2}\eta\right]\right\}^2 (2\xi)^{1/2}\eta^2 \left|\frac{\partial F}{\partial \eta}\right| \tag{22}$$

$$\bar{\epsilon}_0 = 1 + k_2 R_e^{1/2}\,\frac{U_e}{2U_{em}}\,(2\xi)^{1/2} \left\{1 - \operatorname{erf}\,[5(\eta/\eta_\delta - 0.78)]\right\} \int_0^{\eta_\delta} (1-F)\,\mathrm{d}\eta \tag{23}$$

Notice that, as the point of zero skin friction is approached, τ_w tends to zero and A blows up. Yet, in the final expression for the inner eddy viscosity, the exponential simply vanishes and the damping effect disappears.

IV. Comparison with Other Methods

At first it was decided to test the present method against the results of other theoretical methods. Oscillatory turbulent flow over a semi-infinite plate was chosen with an inviscid velocity given by $U_e/U_\infty = 1 + A\cos\omega\tau$, where ω and τ are the dimensionless angular frequency and time. It was assumed that the boundary layer is fully turbulent immediately downstream of the leading edge of the flat plate. The outer flow velocity was assumed to be zero at the origin, that is $\beta = 1$, and the present numerical scheme iterates a few times until it generates a stagnation like profile. The value of β was joined smoothly to its value, after a few steps, as calculated from $\beta = (2\xi/U_e)(\partial U_e/\partial\xi)$. This corresponds to a well-rounded leading edge. The technique is similar to the one employed by McCroskey and Phillipe[20]; it turns out that downstream features of the flow are rather insensitive to the method of starting the calculations. It also should be emphasized that numerical methods, such as the present one, can handle any configuration, and are not confined to simple flows like the flow over a flat plate.

Calculations were performed for a Reynolds number $R_e = 10^7$, for amplitudes $A = 0.125$ and $A = 0.5$, and for angular frequencies of $\omega = 0.157$ and $\omega = 1.57$, in order to compare with the results of Singleton and Nash.[18] Figure 1 shows the variation of the wall shear at a nondimensional distance of $x/L = 1$ for $\omega = 1.57$ and $A = 0.125$. The time is measured in periods $T = 2\pi/\omega$. In the same figure, the results of Kuhn and Nielsen[21] also are shown. The agreement is rather satisfactory if one takes into account the different assumptions incorporated in each method. Singleton and Nash use the turbulent energy equation, carry over three empirical functions from steady flow, and disregard the laminar shear stress. In fact, it is encouraging that their results are in agreement with the other methods, even though their technique cannot be extended to the region next to the wall without another assumption like Townsend's inner law. On the other hand, Kuhn and Nielsen[21] use a Coles velocity profile, again without any adjustments for unsteady flow. The reasonable agreement of the results of the three different methods may be considered as an indication of reliability, but as Nash et al.[17] note, these results should be considered as only numerical experiments, and can be validated only with comparisons with experimental data. All methods indicate a small deviation from the quasisteady results in contrast with the laminar calculations.[32,36] This phenomenon is commented upon in the next section as well, in connection with some extrapolations of Karlsson's[22] experimental data. In Fig. 2 we have plotted the variation of the wall shear for $\omega = 15.7$ and

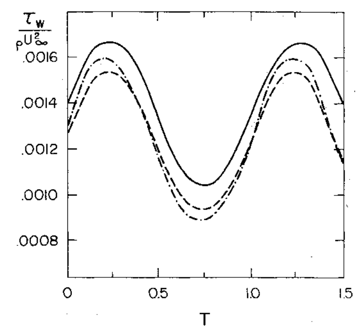

Fig. 1 The wall shear at $x/L = 1$ plotted against time for $\omega = 1.57$ and $A = 0.125$. ---, Singleton and Nash[18]; -·-·- Kuhn and Nielsen[21]; ——present method.

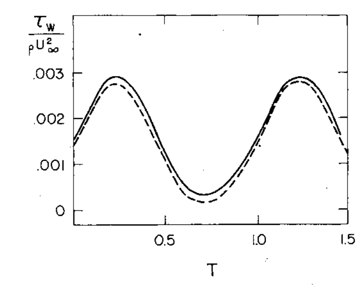

Fig. 2 The wall shear at $x/L = 1$ plotted against time for $\omega = 15.7$ and $A = 0.50$. ---, Singleton and Nash[18]; ——present method.

$A = 0.50$. The method of Ref. 21 is based on a small amplitude hypothesis, and therefore it cannot provide results for this case.

At this point, it is pertinent to comment upon the dependence of the method on mesh configurations and sizes. For turbulent flow, of course, it is necessary to use a variable mesh size in the normal direction, since variations are violent in the neighborhood of the wall and rather mild away from the wall.[35] The authors chose a geometric progress with factors of $\ell = 1.04$ or 1.06 and with increments $\Delta\eta_1 = 0.01$, 0.02, and 0.04, so that the nth element becomes $\Delta\eta_n = \ell^{n-1}\,\Delta\eta_1$. It was proven that the calculations are rather sensitive to the choice of the factor ℓ. The choice of $\Delta\eta_1$ mostly appears to affect the region next to the wall. The scheme of calculations for a specific choice of $\Delta\eta_1$ was very stable, and it required 2 or at most 3 iterations for convergence to the 6th decimal place. The converged value at the wall through differed by about 25 to 30% for calculations with $\Delta\eta_1$ and $\Delta\eta_1/2$ if $\Delta\eta_1$ was of order 0.05. This discrepancy was reduced to approximately 3% for $\Delta\eta_1$ of order 0.001. All the calculations were performed with $\Delta\eta_1 = 0.001$.

The authors also have calculated and compared the displacement thickness, and found that phase variations were in agreement but the values of the quantity itself differed substantially. This was attributed to the fact that the Singleton-Nash calculations start at the origin, with an initial arbitrary thickness of 0.00444, whereas the starting process of our calculations generates a Hiemenz type of profile, with an initial thickness that corresponds to a stagnation-like flow.

V. Comparison with Experimental Results

Unfortunately, there is very little experimental information for time-dependent turbulent boundary layers. This may be attributed partly to the conceptual difficulty of a time-dependent "mean" quantity, which introduces ambiguities, unless the time scale of the forced unsteady effects is different from the time scale of random fluctuations. Karlsson[22] investigated the oscillatory turbulent flow over a flat plate for a wide range of frequencies. Miller[23] carried out similar experiments to measure heat transfer. With the exception of McCroskey and Philippe[20] and Shamroth and Kreskovsky,[14] no other investigator has checked his theoretical predictions against Karlsson's experimental data. The simplicity of the case of a flat plate should in fact provide an ideal test case for comparison. McCroskey and Philippe[20] have compared some phase angle values of displacement thickness and wall shear. Shamroth and Kreskovsky[14] have calculated the fluctuating part of the velocity components. A critique of the relative success of these methods follows, together with the description of the results of the present method.

At first it appears that the average of the velocity profile can be predicted with reasonable accuracy by any of the preceding methods. Following Lighthill,[37] the authors then resolved the unsteady part of the velocity into two components, one in phase with the oscillations of the outer flow and one at a phase difference (advance or delay) of 90° with the outer flow. Assuming that the response of the fluctuating velocity component can be expressed as

$$u_0(x,y,t) = u_1(x,y)\cos(\omega\tau + \phi) + r(x,y,t) \tag{24}$$

where r contains higher harmonics, the averaged products $\overline{u_0 U_\infty A\cos\omega\tau}$ and $\overline{u_0 U_\infty A\cos(\omega\tau + \pi/2)}$ can be calculated. The in-phase and out-of-phase velocity components then can be expressed, respectively, as follows

$$u_{\text{in}}(x,y) = u_1(x,y)\cos\phi = 2^{1/2}\,\overline{u_0\cos\omega\tau}\,(\overline{\cos^2\omega\tau})^{-1/2} \tag{25}$$

$$u_{\text{out}}(x,y) = u_1(x,y)\sin\phi = 2^{1/2}\,\overline{u_0\cos(\omega\tau + \pi/2)}\,(\overline{\cos^2\omega\tau})^{-1/2} \tag{26}$$

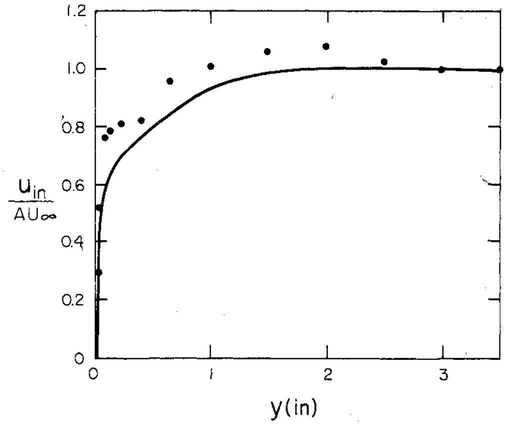

Fig. 3 The in-phase velocity component for $f = \omega/2\pi = 0.33$ and $A = 0.147$. • experimental points[22]; ——present theory.

These calculations now show some discrepancies when compared with Karlsson's data. It appears that the general trend is captured by this model, but the deviation often is unacceptable. For low frequencies, a very mild overshoot of the in-phase component is shown. (Fig. 3). This method is an extension of a scheme for laminar calculations, which is capable of switching on and off the turbulence terms. When the same program is run for laminar flow,[32] it indicates overshoots of the order of 20% or even 30%, which in fact are in good agreement with previous theoretical and experimental results.[37,38]

For large frequencies, the peculiar feature of the overshoot is missing altogether from these results. Shamroth and Kreskovsky,[14] the only other investigators who have attempted similar comparisons, predict a slight overshoot that falls again, considerably below the actual experimental data. Therefore, it is obvious that the Reynolds stress model will have to be modified so that it can predict the steady and the average unsteady velocity amplitudes.

Similar discrepancies seem to appear when one attempts to compare the profile of the phase angle ϕ. Experimental values of ϕ were calculated from the data points of u_{in} and u_{out} of Karlsson[22] according to the formula

$$\phi = \arctan(u_{\text{out}}/u_{\text{in}}) \tag{27}$$

The reader can see in Figs. 4-7 that there is some ambiguity, especially in the region next to the wall, which perhaps is attributed to the fact that measurements were taken with a hot-wire anemometer. For large frequencies, the phase angle appears to increase sharply next to the wall, perhaps approaching a value not far from 45°, a behavior reminiscent of laminar flow.[32,37,38] For smaller frequencies, however, one cannot discard a clear tendency of the experimental points to turn around and decrease again, meeting the wall with a positive slope. The present method seems to predict the behavior for large frequencies reasonably well, but is unable to simulate the trend for lower frequencies.

In Fig. 8, the authors have plotted the theoretical prediction and experimental values of the phase angle at the wall vs frequency. This, of course, is the phase angle of the skin friction as well, and it is observed again, as in laminar flow, that the flow next to the wall is always in advance with respect to the outer flow. The theoretical points were calculated with the Cebeci and Smith correction for pressure gradients. McCroskey and Philippe[20] also have plotted the wall shear phase against frequency, but for very small values of the nondimensional angular frequency. Nevertheless, their results seem to attain an asymptotic value of less than 15°. Singleton and Nash[18] also found wall shear phase angles of that order and even smaller. The authors believe that the experimental evidence of Karlsson indicates that, even for turbulent flow, one should expect phase angles of the order of 30° or 40°.

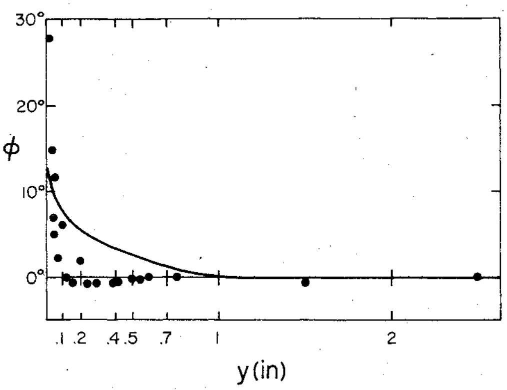

Fig. 4 Velocity fluctuation phase angles for $f=\omega/2\pi=7.65$ and $A=0.127$. experimental points[22]; ——present theory.

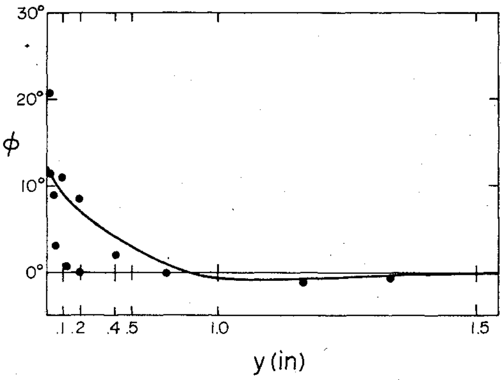

Fig. 5 Velocity fluctuation phase angles for $f=\omega/2\pi=4$, $A=0.062$. • experimental points[22]; ——present theory.

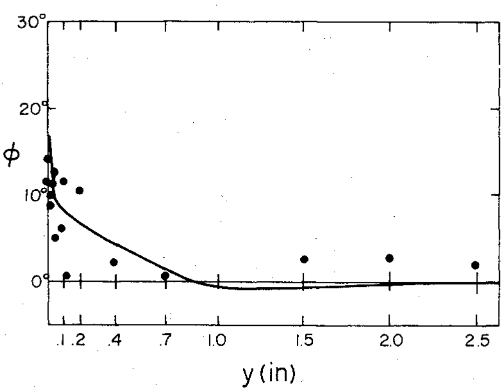

Fig. 6 Velocity fluctuation phase angle for $f=\omega/2\pi=4$, $A=0.136$. • experimental points[22]; ——present theory.

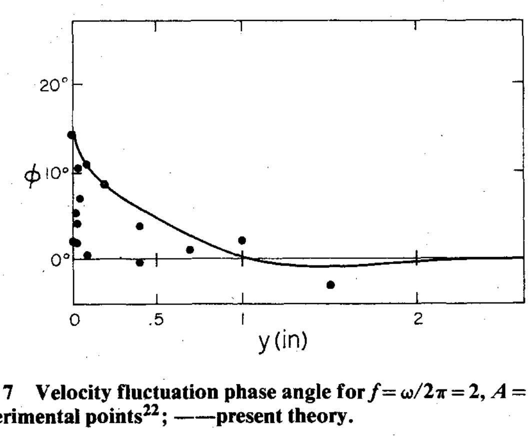

Fig. 7 Velocity fluctuation phase angle for $f=\omega/2\pi=2$, $A=0.136$. • experimental points[22]; ——present theory.

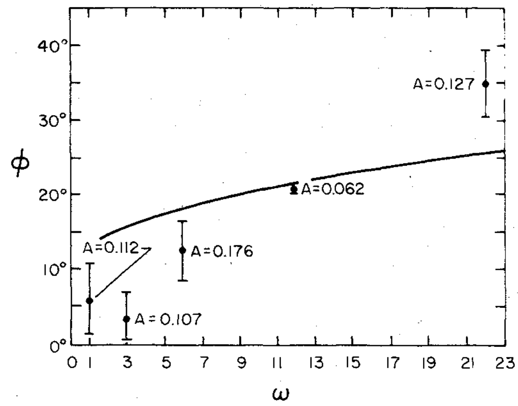

Fig. 8 Wall shear phase angle as a function of dimensionless angular frequency ω. Experimental data from Ref. 22.

Physically, one would tend to attribute the sharp increase of the phase angle next to the wall to the fact that the inner region of the turbulent boundary layer, the so-called "viscous sublayer," is not far from a laminar layer, and hence it ought to behave like Lighthill's laminar layers.[37] However, the authors believe that this tendency is initiated far from the wall, at a distance of at least one order of magnitude larger than the thickness of the viscous sublayer.

VI. A Transient Flow with Separation

In order to avoid dynamic effects at the initial x,y plane, a transient outer flow distribution was chosen, having vanishing time derivatives at time $t=0$. In this way, the calculation for $t=0$ sweeps the x,y plane with $\partial U_e/\partial t=0$, and generates a steady-state flow pattern. For simplicity, and with no less of generality, a linearly deccelerating outer flow was chosen that forces the turbulent boundary layer to separate at a station S_I. Separation is signaled by the vanishing of the skin friction,[28,29] as well as by the familiar properties of the separation singularity.[29] In particular, quantities like $\partial u/\partial x$, v, δ, etc. increase sharply as separation is approached, and eventually convergence becomes impossible and the computation is interrputed.

At this point a comment about the possible removal of the singularity perhaps is necessary. In recent years many investigators have removed the vanishing-skin-friction singularity by prescribing, in an arbitrary manner, the boundary-layer thickness, the wall shear, etc. (for more details and references see Ref. 36) instead of the outer flow velocity U_e. In this way, they were able to proceed through the point of vanishing wall shear into a region of reversed flow. This technique is justified, of course, only if the recirculating bubble is thin and embedded in the boundary layer so that the boundary layer approximation still is valid. If the point of zero skin friction marks the initiation of a turbulent wake and a sharp breakaway of the thin viscous layer from the wall, then it would be senseless to insist upon using the boundary layer beyond this point. Nevertheless, the heuristic methods described previously probably could remove the singularity again, and continue calculating an imaginary flowfield attached to the wall which has no counterpart in real life. The present authors maintain that the separation-singularity is not just a mathematical pecularity, but the response of the boundary layer, in a neighborhood in which the actual flow separates from the wall. This point of view was substantiated by showing that, for steady laminar flow over moving walls, the singularity does not appear at the point of zero wall shear.[27] Further studies of unsteady laminar flows indicated experimentally that the point of vanishing wall shear is not related to separation,[26] and numerically that the same point is not accompanied by a singularity,[25,31] two facts that are equivalent according to the theory of Refs. 24 and 27. The

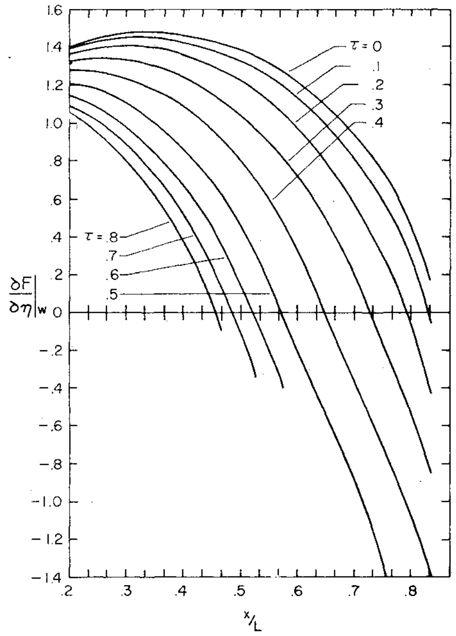

Fig. 9 The wall shear function parameterized with time for a transient variation of the outer flow, as given by Eq. (28)

numerical calculations indicated that a separation singularity appears downstream in a neighborhood where separation was expected. In the present paper we have investigated the possibility of extending the previous work to turbulent flows. For a transient flow that corresponds to an upstream motion of separation, the authors studied the intermediate time steps in detail and verified that the point of vanishing shear is not a singular point, even in turbulent flow, a fact already reported by Nash, Carr, and Singleton.[17] There are no experimental data, but in the spirit of Ref. 24 and the experience gathered with laminar flows the authors conclude that the point of vanishing shear is not related to separation in unsteady turbulent boundary layers. This implies that the calculation should proceed into a region of partially reversed flow, and hence in the opposite direction of integration of the parabolic equation. This was accomplished by an upwind differencing scheme, which the authors have used previously[9,25,31] for laminar flows.

Figure 9 shows the variation of the skin friction for various time steps τ, for an outer flow distribution given by

$$U_e/U_\infty = 1-(\tfrac{1}{2}+\tau^2)x/L \quad \text{for } \tau \leq \tfrac{1}{2}$$

$$U_e/U_\infty = 1-(2\tau+\tau^2)x/L \quad \text{for } \tfrac{1}{2} \leq \tau \leq 1$$

$$U_e/U_\infty = 1-x/L \quad \text{for } \tau \geq 1 \tag{28}$$

For $t=0$, $\partial U_e/\partial t=0$, a steady-state flow is generated and the steepening of the shear slope as the point $\tau_w=0$ is approached is an indication of separation.[27] For later times, the calculation proceeds through the point of zero skin friction with no evidence of singularity, which also is indicated in the figure with a finite slope of the curve at the point of $\tau_w=0$. The curves then are terminated at a point at which the familiar features of the separation singularity appear. This behavior is not so pronounced in the figure but a more careful study of other properties of the flow unambiguously define the station of the singularity.[29] Finally, the wall shear variation approaches the steady-state conditions that correspond to $U_e = U_\infty\ (1-x/L)$ steady separation, accompanied by an infinite slope of the skin friction, sppears at S_2.

VII. Conclusions and Recommendations

The main goal of the present work is to extend the classical eddy viscosity and mixing length models, with all of their recent refinements, to time-dependent flows. This is done by a straightforward generalization of all the appearing quantities, whereby, for example, the pressure gradient is augmented by a time derivative, and the instantaneous values of the outer flow velocity, displacement thickness, etc., are used to calculate the eddy viscosity, and the constants of the model are given the values estimated by comparison to experiments of steady flow. This approach certainly is only a first step in estimating the possibility of calculating time-dependent turbulent flows with approximate models, and has been followed by previous investigators who have used various other approximate methods. If is of course possible that the forced oscillations of the outer flow interact with the random fluctuations of the turbulent motion. If then energy is transferred from the mean to the turbulent motion, it is most unlikely that the empirical models for eddy viscosity can be carried over from steady to unsteady flow.

The present method appears in principle to have some advantages and disadvantages over the other methods, but the results, at least as far as the skin friction is concerned, seem to be in reasonable agreement. It appears further that very little effort has been directed in comparing the theoretical predictions with experimental data. True, the available experimental data are minimal and new experiments are badly needed. Yet it seems a little premature to undertake extensive engineering calculations using models whose validity is questioned still.

The present work presents a detailed comparison with the available experimental information. The fluctuating velocity components are found to be predicted correctly qualitatively, but not quantitatively, a phenomenon encountered also in Ref. 14 (the only other effort to test these quantities). The phase angle distribution is predicted with reasonable accuracy. The present theory, unlike all other methods and the experimental data, indicates that the wall shear angles may reach, for large frequencies, values of the order of 20° to 30°. The authors feel that approximate models should be improved to predict all of the detailed features of unsteady turbulent flow, especially when it is expected that such models will be used to predict separation.

In the present paper it also was indicated that the point of vanishing wall shear is not singular for unsteady turbulent flow, and the integration proceeds through a region of reversed flow. Separation was signaled by some familiar properties of the singularity that appears at the point of zero wall shear for steady flow and downstream of this point in the case of upstream moving separation.

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